Explainable and Computationally Efficient Decision Making with Quantitative Abstract Argumentation Frameworks

Tutorial at the 43rd German Conference on Artificial Intelligence (KI 2020)

Nico Potyka
Schedule

Block 1 (8:30 – 10:30)
- Overview
- Probabilistic Epistemic Argumentation Overview and Applications
- Gradual Bipolar Argumentation Overview and Applications
- Probabilistic Epistemic Argumentation Advanced Topics

Block 2 (11:00-13:00)
- Gradual Bipolar Argumentation Advanced Topics

Small Breaks in between
Questions and comments are very welcome at any time
A (Biased) Glimpse of Abstract Argumentation

From Dung Frameworks to Quantitative Bipolar Argumentation
Abstract vs Structured Argumentation

**Structured Argumentation**
- model internal structure of arguments

**Abstract Argumentation**
- Abstract from content, focus on relationships

Structure can be important for NLP and legal reasoning

See Tutorial T7 today from 14:30 – 18:30 to learn more

Sufficient for many „decision problems“
• „[...] a statement is believable if it can be argued successfully against attacking arguments.” [1]
• **Given** argumentation framework (edges = attacks),

• **Decide** which arguments can be accepted

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Bipolar Abstract Argumentation

- **Shortcoming**: we do not only consider *contra*, but also *pro* arguments
- **Bipolar abstract argumentation** adds support edges [3]

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Quantitative Bipolar Abstract Argumentation

- **Shortcoming**: often, we do not completely accept or reject arguments. Rather, we tend to accept or tend to reject arguments gradually.

- **Quantitative frameworks** evaluate arguments numerically.
Two Interesting Quantitative Approaches

**Probabilistic Epistemic Argumentation**
- Evaluation: probabilities
- Complexity: (polynomial)
- Model
  - Bipolar Argumentation Graph
  - Semantical Constraints

**Implementation**
https://sourceforge.net/projects/probabble/

**Gradual Bipolar Argumentation**
- Evaluation: strength values
- Complexity: (polynomial)
- Model
  - Bipolar Argumentation Graph
  - Initial Weights
  - Update function

**Implementation**
https://sourceforge.net/projects/attractorproject/


Probabilistic Epistemic Argumentation

Basics and Some Applications
Probabilistic Epistemic Argumentation

• Ingredients
  • BAG
  • Semantical Constraints like
    • Founded: If A unattacked, then \( P(A) \geq 0.5 \)
    • Coherence: If A attacks B, then \( P(B) \leq 1 - P(A) \)
    • S-Coherence: If A supports B, then \( P(A) \leq P(B) \)
    • ...

• If all constraints are „linear atomic“, solvable in polynomial time [4]
  \[
  \sum_{i=1}^{n} c_i \cdot P(A_i) \leq c_0
  \]

**Application: Decision Making**

- **T₁: Immobilization of Limb**
  - **A₁:** Immobilization will result in atrophy and make future injuries more likely
  - **S₁:** Study 1

- **T₂: Immediate Physical Therapy**
  - **A₂:** Immobilization will prevent further injury and thus speed up healing
  - **S₂:** Study 2

- **A₃:** Mobilization will cause additional stress and thus slow down healing
  - **S₃:** Study 3

- **A₄:** Mobilization will improve blood circulation and thus speed up healing
  - **S₄:** Study 4

**Notes:**
- **T₁** (Immobilization of Limb)
  - **A₁:** Immobilization will result in atrophy and make future injuries more likely
  - **S₁:** Study 1
- **T₂** (Immediate Physical Therapy)
  - **A₂:** Immobilization will prevent further injury and thus speed up healing
  - **S₂:** Study 2

**Stress and Healing:**
- **A₃:** Mobilization will cause additional stress and thus slow down healing
  - **S₃:** Study 3

**Blood Circulation and Healing:**
- **A₄:** Mobilization will improve blood circulation and thus speed up healing
  - **S₄:** Study 4
$T_1$: Immobilization of Limb

$T_2$: Immediate Physical Therapy

$A_1$: Immobilization will result in atrophy and make future injuries more likely

$A_2$: Immobilization will prevent further injury and thus speed up healing

$A_3$: Mobilization will cause additional stress and thus slow down healing

$A_4$: Mobilization will improve blood circulation and thus speed up healing

$S_1$: Study 1

$S_2$: Study 2

$S_3$: Study 3

$S_4$: Study 4

$S_5$: Study 5

$P(B) \leq 1 - P(A)$

$P(B) \geq P(A)$
\[ \{P(S_i) \geq 0.5 \mid i = 1, \ldots, 5\} \]
\[
\{ P(S_i) \geq 0.5 \mid i = 1, \ldots, 5 \} \begin{bmatrix}
0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5
\end{bmatrix}
\]

\[
\{ P(S_1) = 1 \} \begin{bmatrix}
0 & [0,1] & 1 & 0 & [0,1] & [0,1] & 1 & 0 & [0,1] & [0,1]
\end{bmatrix}
\]
Application: Belief Consolidation

Application: Computational Persuasion

Since you do little excercise, you should do a regular excercise class
When I do exercise, I get very hungry and I put on weight
When I do exercise, I get hungry and I put on weight.
(User goal) You want to stay healthy for your children.

(Future risk) If you don’t do more exercise, you will have a raised risk of health problems.

(Emotional) Many of your colleagues are doing an exercise class and so you will feel left out if you don’t participate in one.

(Persuasion goal) To improve your health, you could join an exercise class.

(Opinion) Exercise classes are boring.

(Opinion) There are exciting exercise classes.

(Example) You can do an indoor rock climbing class.
You want to stay healthy for your children.
If you don’t do more exercise, you will have a raised risk of health problems.
User Model

<table>
<thead>
<tr>
<th>Argument</th>
<th>Belief</th>
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<tr>
<td>A1</td>
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<td>Goal</td>
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User Update in Dialogue

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<td>Goal</td>
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Posit A2
Gradual Bipolar Argumentation

• Ingredients
  • BAG
  • Initial Weights
  • Update Function

• In many interesting cases, solvable in polynomial time [3]

Computing Final Strength Values
A: The Wonder-Phone is the best new generation phone

B: No, the Magic-Phone is the best new generation phone

C: links to a review of the Magic-Phone giving poor scores due to bad battery performance

D: C is ignorant, since subsequent reviews noted only one of the first editions had such problems: [links].

E: D is wrong. I found C knows about that but withheld the information. Here's a link to another thread proving it.

A: The Wonder-Phone is the best new generation phone

B: No, the Magic-Phone is the best new generation phone

C: links to a review of the M-Phone giving poor scores due to bad battery performance

D: C is ignorant, since subsequent reviews noted only one of the first editions had such problems: [links].

E: D is wrong. I found C knows about that but withheld the information. Here’s a link to another thread proving it.
Decision Support (Rago et al. 2016)

Issue: How to spend council’s budget?

A1: Build a new cycle path.
A2: Repair current infrastructure.

P1: Cyclists complain of dangerous roads.
P2: A path would enhance the council’s green image.
P3: Potholes have caused several accidents recently.
C1: Significant disruptions to traffic would occur.
C2: Environmentalists are a fraction of the population.
C3: Recent policies already enhance this green image.
C4: Donors do not see the environment as a priority.
Explainable Review Aggregation (Cocarascu et al. 2019)

From Reviews to BAGs

Pipeline

1. Extract argumentative phrases.
2. Associate phrases with features.
3. Compute sentiment of arguments.
4. Aggregate sentiments and derive attack/support relations and weights.

"Spotlight", the Oscar-winning movie of two years ago, made me feel proud to be a journalist. The Post, which I finally saw over the weekend, reminded me how much fun the business is. It was once upon a time. I'm pretty sure it still has its moments.

Sometimes casting is everything. A city room is a collection of characters, and the most efficient way to put that across in a movie is the way director Steven Spielberg chose here: bring together a bunch of your favorite character actors and let them have at it. The degree of permissible overacting set by Meryl Streep, who as Kay Graham turns in the kind of bravura performance that you never for a minute forget is all technique. You watch her do a scene and want to hold up a board that says "10." Just about every role in the movie of any consequence is played by someone we know from somewhere else and are delighted to see again. Like Matthew Rhys from The Americans, and Bob Odenkirk and Jesse Plemons from Breaking Bad, and Tracy Letts from Homeland and Lady Bird, and his wife, Carrie Coon, from Fargo. And Michael Stuhlbarg.
Explainable Review Aggregation (Cocarascu et al. 2019)

Explainable Review Aggregation (Cocarascu et al. 2019)

Then, a simple argumentation dialogue is such that for any \( \alpha \in \mathcal{A} \):

if \( \alpha = m \) and \( \sigma(\alpha) < 0.6 \) and \( \exists \beta \in \mathcal{L}^-(\alpha) \cup \mathcal{L}^+(\alpha) \) s.t. \( \sigma(\beta) > 0 \):

\begin{align*}
Q(\alpha) &= \{ \text{Why was } \alpha \text{ poorly rated?} \} \\
\mathcal{X}(\alpha) &= \{ \text{This movie was poorly rated} \} + \\
&\quad \{ 0.6 \text{max}(\mathcal{L}^-(m)) \} + \{ 0.4 \text{max}(\mathcal{L}^+(m)) \}; \quad \text{else}
\end{align*}

if \( \alpha = m \) and \( \sigma(\alpha) \geq 0.6 \) and \( \exists \beta \in \mathcal{L}^-(\alpha) \cup \mathcal{L}^+(\alpha) \) s.t. \( \sigma(\beta) > 0 \):

\begin{align*}
Q(\alpha) &= \{ \text{Why was } \alpha \text{ highly rated?} \} \\
\mathcal{X}(\alpha) &= \{ \text{This movie was highly rated} \} + \\
&\quad \{ 0.6 \text{max}(\mathcal{L}^-(m)) \} + \{ 0.4 \text{max}(\mathcal{L}^+(m)) \}; \quad \text{else}
\end{align*}

if \( \alpha \in \mathcal{F} \) and \( V^+(\alpha) < V^-(\alpha) \) and \( \exists \beta \in \mathcal{L}^-(\alpha) \cup \mathcal{L}^+(\alpha) \) s.t. \( \sigma(\beta) > 0 \):

\begin{align*}
Q(\alpha) &= \{ \text{Why was/were the } \alpha \text{ considered to be poor?} \} \\
\mathcal{X}(\alpha) &= \{ \text{The } \alpha \text{ was/were considered to be poor} \} + \\
&\quad \{ 0.6 \text{max}(\mathcal{L}^-(m)) \} + \{ 0.4 \text{max}(\mathcal{L}^+(m)) \}; \quad \text{else}
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\begin{align*}
Q(\alpha) &= \{ \text{Why was/were the } \alpha \text{ considered to be great?} \} \\
\mathcal{X}(\alpha) &= \{ \text{The } \alpha \text{ was/were considered to be great} \} + \\
&\quad \{ 0.6 \text{max}(\mathcal{L}^-(m)) \} + \{ 0.4 \text{max}(\mathcal{L}^+(m)) \}; \quad \text{else}
\end{align*}

if \( V^+(\alpha) < V^-(\alpha) \) and \( \exists \beta \in \mathcal{L}^-(\alpha) \cup \mathcal{L}^+(\alpha) \) s.t. \( \sigma(\beta) > 0 \):

\begin{align*}
Q(\alpha) &= \{ \text{What did critics say about the } \alpha \text{ being poor?} \} \\
\mathcal{X}(\alpha) &= \{ p \text{ from c constituting } V(c, \alpha) = -1 \}; \quad \text{else}
\end{align*}

if \( V^+(\alpha) \geq V^-(\alpha) \) and \( \exists \beta \in \mathcal{L}^-(\alpha) \cup \mathcal{L}^+(\alpha) \) s.t. \( \sigma(\beta) > 0 \):

\begin{align*}
Q(\alpha) &= \{ \text{What did critics say about the } \alpha \text{ being great?} \} \\
\mathcal{X}(\alpha) &= \{ p \text{ from c constituting } V(c, \alpha) = +1 \}.
\end{align*}
Explainable Review Aggregation (Cocarascu et al. 2019)

user: Why was Phantom Thread highly rated?
ADA: This movie was highly rated because the acting was great.
user: Why was the acting considered to be great?
ADA: The acting was considered to be great because Daniel Day-Lewis was great.
user: What did critics say about Daniel Day-Lewis being great?
ADA: “Daniel Day-Lewis remains our greatest actor...”

Quantitative AAFs assign numerical values (belief, strength) to arguments. Values can be explained by going backwards through the graph. Rules can be used to generate simple automated explanation dialogues.
Summary

- Probabilistic Epistemic Argumentation
  - Evaluation: probabilities
  - Complexity: (polynomial)
- Model
  - Bipolar Argumentation Graph
  - Semantical Constraints
- Implementation
  https://sourceforge.net/projects/probabble/

- Gradual Bipolar Argumentation
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Probabilistic Epistemic Argumentation

Semantics and Computation
Probabilistic Epistemic Argumentation

• Ingredients
  • BAG
  • Semantical Constraints like
    • Founded: If A unattacked, then $P(A) \geq 0.5$
    • Coherence: If A attacks B, then $P(B) \leq 1 - P(A)$
    • S-Coherence: If A supports B, then $P(A) \leq P(B)$
    • ...

• If all constraints are „linear atomic“, solvable in polynomial time [2]

$$\sum_{i=1}^{n} c_i \cdot P(A_i) \leq c_0$$

Some Other Linear Atomic Constraints

\[ P(CE) \geq 0.3 \cdot P(M) + 0.3 \cdot P(O) \]
\[ \equiv \]
\[ 0.3 \cdot P(M) + 0.3 \cdot P(O) - P(CE) \leq 0 \]

\[ P(M) \geq \max(0.9 \cdot P(V_1), 0.9 \cdot P(D_1)) \]
\[ \equiv \]
\[ 0.9 \cdot P(V_1) - P(M) \leq 0 \]
\[ 0.9 \cdot P(D_1) - P(M) \leq 0 \]
Epistemic States (Probability Functions)

- $P(A) = 0.6$
- $P(B) = 0.3$
- $P(C) = 0.7$

- $P(B) \leq 1 - P(A)$?
- $P(C) \geq P(B)$?

Consistent constraints are usually satisfied by an infinite number of probability functions
Reasoning Problems

• **SATISFIABILITY:**
  - Given BAG with Semantical Constraints
  - Is there a probability function that satisfies all constraints?

• **ENTAILMENT**
  - Given BAG with satisfiable Semantical Constraints and argument A
  - Compute tight lower and upper bounds for $P(A)$

• If all constraints are „linear atomic“, both problems can be solved efficiently [2]

## Probability Labellings

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<th>B</th>
<th>C</th>
<th>P(A,B,C)</th>
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Lemma: The following statements are equivalent:

1. $P$ satisfies a \textit{linear atomic constraint} $c$.
2. All $P'$ in $[P]$ satisfy $c$.
3. $L = r([P])$ satisfies $c$.

Solving Reasoning Problems

SATISFIABILITY and ENTAILMENT problem can be encoded as LPs

- Interior-point methods give polynomial worst-case bound
- Simplex algorithm is often faster in practice (even though worst-case runtime is exponential)

Adding Expressiveness

If $P \neq NP$, we lose polynomial runtime guarantees when allowing

- Disjunctions of arguments $P(A \lor B)$
- Conjunctions of arguments $P(A \land B)$

Some problems remain tractable when applying the principle of maximum entropy

- but results in strong independency assumptions
- constraints like Coherence and S-Coherence can still give some meaningful guarantees for relationships between arguments

 Updates Revisited

![Diagram](image)

### Argument Belief

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![Diagram](image)

### Argument Belief

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**Posit A2**
**Definition 1 (Epistemic Update Operator).** An epistemic update operator is a function $\mathcal{U} : P_A \times C_A \rightarrow P_A \cup \{\bot\}$ that satisfies the following properties:

- **Success:** If $C \subseteq C_A$ is satisfiable, then $\mathcal{U}(P, C) \in \text{Sat}_P(C)$.
- **Failure:** If $C \subseteq C_A$ is not satisfiable, then $\mathcal{U}(P, C) = \bot$.
- **Representation Invariance:** If $C_1 \equiv C_2$, then $\mathcal{U}(P, C_1) = \mathcal{U}(P, C_2)$.
- **Idempotence:** If $C \subseteq C_A$ is satisfiable, then $\mathcal{U}(\mathcal{U}(P, C), C) = \mathcal{U}(P, C)$.

Update operators can be defined by minimizing distance between prior and new epistemic state.
1. Minimize atomic *LS distance* to $P$ (solution may not be unique)
2. Minimize *LS distance* to $P$ (solution is unique)

Minimize *LS distance* to $L$ (solution is unique)
Theorem 1. Let \( C \subseteq C_A \) be a finite and satisfiable set of linear atomic constraints and let \( L \in \mathcal{L}_A \). Then \( LU^2_X(L, C') = L^* \) is well-defined and can be computed in polynomial time. Furthermore, \( L^* = r([U^2_A_t(P, C)]) \) for all \( P \in r^{-1}(L) \).
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\[
P(\text{Goal}) \leq 1 - 0.5 \cdot P(A1) - 0.5 \cdot P(A3)
\]

\[
P(A1) \leq 1 - P(A2)
\]

\[
P(\text{Goal}) \geq P(A2)
\]
Do you agree that A2?

- Strongly Agree: $P(A2) \geq 0.9$
- Agree: $P(A2) \geq 0.7$
- Indifferent
- Disagree: $P(A2) \leq 0.3$
- Strongly Disagree: $P(A2) \leq 0.1$

$P(Goal) \leq 1 - 0.5 \times P(A1) - 0.5 \times P(A3)$

- $P(A1) \leq 1 - P(A2)$
- $P(Goal) \geq P(A2)$

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</tr>
<tr>
<td>A3</td>
<td>0.2</td>
</tr>
<tr>
<td>Goal</td>
<td>0.7</td>
</tr>
</tbody>
</table>

\[ P(\text{Goal}) \leq 1 - 0.5 \times P(\text{A1}) - 0.5 \times P(\text{A3}) \]

\[ P(\text{A1}) \leq 1 - P(\text{A2}) \]

\[ P(\text{A2}) \geq 0.7 \]

\[ P(\text{Goal}) \geq P(\text{A2}) \]
<table>
<thead>
<tr>
<th>Argument</th>
<th>Belief</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.8</td>
</tr>
<tr>
<td>A2</td>
<td>0.1</td>
</tr>
<tr>
<td>A3</td>
<td>0.2</td>
</tr>
<tr>
<td>Goal</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Diagram:
- A1 is connected to Goal and A3.
- A2 is connected to A1 andGoal.
- Dotted line indicates A2 is connected to A3.
Do you agree that if A2 then not A1?

- **Strongly Agree**: $P(A1) \leq 1 - P(A2)$
- **Agree**: $P(A1) \leq 1 - 0.5 \cdot P(A2)$
- **Indifferent**
- **Disagree**
- **Strongly Disagree**
<table>
<thead>
<tr>
<th>Argument</th>
<th>Belief</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.8</td>
</tr>
<tr>
<td>A2</td>
<td>0.1</td>
</tr>
<tr>
<td>A3</td>
<td>0.2</td>
</tr>
<tr>
<td>Goal</td>
<td>0.2</td>
</tr>
</tbody>
</table>

\[ P(A1) \leq 1 - P(A2) \]
Probabilistic Epistemic Argumentation

• Ingredients
  • BAG
  • Semantical Constraints like
    • Founded: If A unattacked, then \( P(A) \geq 0.5 \)
    • Coherence: If A attacks B, then \( P(B) \leq 1 - P(A) \)
    • S-Coherence: If A supports B, then \( P(A) \leq P(B) \)
    • ...

• If all constraints are „linear atomic“, solvable in polynomial time [2]

\[
\sum_{i=1}^{n} c_i \cdot P(A_i) \leq c_0
\]

Gradual Bipolar Argumentation

Semantics
Development of new phone was too expensive. They will have to cut down R&D and will not stay competitive in future.
The new phone is innovative and will increase profits considerably.
Investment in R&D is far beyond competitors' investment. Company is likely to become market leader.
Buy: 0.5

A2: 0.7

A1: 0.8

A3: 0.9

Sell: 0.5
Weighted Bipolar Argumentation Graph (BAG)

- Set of arguments
- Initial weights
- Attack and support relation

**Semantics:** define final strength of arguments based on

- Initial weights and
- Strength of parents

\[ s(i) = f(w(i), \text{Parents}(i)) \]
<table>
<thead>
<tr>
<th></th>
<th>Buy: 0.5</th>
<th>Sell: 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A2</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>
Computing Strength Values in Acyclic BAGs

- Compute topological ordering
- Evaluate arguments in order

\[ s(i) = f(w(i), \text{Parents}(i)) \]

Computing Strength Values in Cyclic BAGs

- Set initial strength values to initial weights
- Update by applying update formula to all arguments simultaneously
- Repeat until process converges
Modular Semantics *(Mossakowski, Neuhaus 2018)*

- Similar ideas have been considered before
  - Local Gradual Valuations (Amgoud et al. 2008)
  - Semantic Frameworks (Leite, Martins 2011)

DF-QuAD

\[ a = \prod_{i \in \text{Att}(B)} (1 - s_i) - \prod_{i \in \text{Sup}(B)} (1 - s_i) \]

\[ s = \begin{cases} 
  w + w \times a & \text{if } a < 0 \\
  w + (1 - w) \times a & \text{else}
\end{cases} \]

Some Special Cases: No Parents

- Aggregation: $a = 1 - 1 = 0$
- Influence: $s = w$
Some Special Cases: No Supporters

- **Aggregation:** 
  \[ a = \prod_{i \in \text{Att}(B)} (1 - s_i) - 1 \leq 0 \]

- **Influence:** 
  \[ s = w + w \times a \leq w \]
Some Special Cases: No Attackers

- **Aggregation**: \( a = 1 - \prod_{i \in \text{sup}(B)} (1 - s_i) \geq 0 \)

- **Influence**: \( s = w + (1 - w) \times a \geq w \)
Euler-based Semantics

**Aggregation:**
\[ a = \sum_{i \in Sup(B)} S_i - \sum_{i \in Att(B)} S_i \]

**Influence:**
\[ s = 1 - \frac{1 - w^2}{1 + w \times e^a} \]

Quadratic-energy Model

**Aggregation:**
\[ a = \sum_{i \in \text{Sup}(B)} s_i - \sum_{i \in \text{Att}(B)} s_i \]

**Influence:**
\[ s = \begin{cases} 
  w + (1 - w) \times \frac{a^2}{1 + a^2} & \text{if } a > 0 \\
  w - w \times \frac{a^2}{1 + a^2} & \text{else} 
\end{cases} \]

Aggregation Functions

- **Product:** $\prod_{i \in \text{Att}(B)} (1 - s_i) - \prod_{i \in \text{Sup}(B)} (1 - s_i)$

- **Sum:** $\sum_{i \in \text{Sup}(B)} s_i - \sum_{i \in \text{Att}(B)} s_i$

- **Top:** $\max \{s_i : i \in \text{Sup}(B)\} - \max \{s_i : i \in \text{Att}(B)\}$
Influence Functions

• **Linear**($k$):
  
  \[
  \begin{cases} 
  w + \frac{w}{k} \times a & \text{if } a < 0 \\
  w + \frac{1-w}{k} \times a & \text{else}
  \end{cases}
  \]

• **Euler-based**:
  
  \[
  1 - \frac{1 - w^2}{1 + w \times e^a}
  \]

• **$q_{\text{max}}(k)$**:
  
  \[
  \begin{cases} 
  w + \frac{1-w}{k} \times \frac{a^2}{1+a^2} & \text{if } a > 0 \\
  w - \frac{w}{k} \times \frac{a^2}{1+a^2} & \text{else}
  \end{cases}
  \]
Semantical Desiderata

- Equivalence
- Neutrality
- Dummy
- Maximality/Minimality
- Strengthening/Weakening

- Void Precedence
- Triggering
- Counting
- Proportionality
- ...

(Baroni et al. 2018) showed that most properties can be broken down to two fundamental principles called *Balance* and *Monotonicity*.

Balance (Intuition)

1. If attackers and supporters are "equally strong", strength should be equal to initial weight

2. If attackers are "stronger (weaker) than" supporters, strength should be smaller (larger)
Balance: DF-QuAD

- **Aggregation:** $a = (1 - 1) - (1 - 1) = 0$
- **Influence:** $s = 0.5 + (1 - 0.5) \times 0 = 0.5$
Balance: DF-QuAD

- **Aggregation:** $a = (1 - 0.8) - (1 - 1) = 0.2$

- **Influence:** $s = 0.5 + (1 - 0.5) 	imes 0.2 = 0.6$
Balance: DF-QuAD

- **Aggregation**: \( a = (1 - 1) - (1 - 1) \times (1 - 1) = 0 \)
- **Influence**: \( s = 0.5 + (1 - 0.5) \times 0 = 0.5 \)

*Product Aggregation and Top Aggregation can violate balance*
Monotonicity (Intuition)

1. If the „same impact“ (in terms of initial weight, attack and support) acts on A1 and A2, then they should have the same strength.

2. If the impact on A1 is „more positive“, then it should have a larger strength than A2.
Monotonicity: Euler-based Semantics

\[ a = -0.5 \]

\[ s = 1 - \frac{1 - 0.5^2}{1 + 0.5 \times \exp(-0.5)} \approx 0.42 \]

\[ a = -1 \]

\[ s = 1 - \frac{1 - 0.5^2}{1 + 0.5 \times \exp(-1)} \approx 0.37 \]
Monotonicity: Euler-based Semantics

Euler-based Influence violates monotonicity in boundary cases

- $a = -0.5$
  
  $s = 1 - \frac{1-(-0.5)^2}{1 + 1 \times \exp(-0.5)} = 1$

- $a = -1$
  
  $s = 1 - \frac{1-(-1)^2}{1 + 1 \times \exp(-1)} = 1$
Beyond Balance and Monotonicity (AAMAS 2019)

• **Duality:** Attack and support should behave „in a dual manner“

• **Open-Mindedness:** strength should become arbitrarily close to 0 (1) if we keep adding „strong“ attackers (supporters)

Duality: DF-QuAD

• $a = (1 - 0.8) - 1 = -0.8$

• $s = 0.5 - 0.5 \times 0.8 = 0.1$

• $a = 1 - (1 - 0.8) = 0.8$

• $s = 0.5 + (1 - 0.5) \times 0.8 = 0.9$
Duality: Euler-based

Euler-based Influence can violate Duality

A: 0.8

B: 0.5

0.39 (-0.11)

- \(a = -0.8\)
- \(s = 1 - \frac{1 - 0.5^2}{1 + 0.5 \times \exp(-0.8)} = 0.39\)

C: 0.8

D: 0.5

0.65 (+0.15)

- \(a = 0.8\)
- \(s = 1 - \frac{1 - 0.5^2}{1 + 0.5 \times \exp(0.8)} = 0.65\)
Open-Mindedness: DF-QuAD

- A: 1
  - B: 1
    - 0
  - 0
    - ✓

- C: 1
  - D: 0
    - 1
    - ✓

\[
\begin{align*}
\text{a} &= (1 - 1) - 1 = -1 \\
\text{s} &= 1 - 1 \times 1 = 0
\end{align*}
\]

\[
\begin{align*}
\text{a} &= 1 - (1 - 1) = 1 \\
\text{s} &= 0 + (1 - 0) \times 1 = 1
\end{align*}
\]
Open-Mindedness: DF-QuAD

\[ a = 0 - 0 = 0 \]
\[ s = 0.5 - 0.5 \times 0 = 0 \]

\[ a = 0 - 0 = 0 \]
\[ s = 0.5 + (1 - 0.5) \times 0 = 0.5 \]
Open-Mindedness: Euler-based

\[ a = -n \rightarrow -\infty \]

\[ s = 1 - \frac{1 - 0.5^2}{1 + 0.5 \times \exp(-n)} > 0.25 \]

Euler-based Influence can violate Open-Mindedness
## Summary: Potential Semantical Problems

<table>
<thead>
<tr>
<th>Aggregation Function</th>
<th>Balance</th>
<th>Monotonicity</th>
<th>Duality</th>
<th>Open-Mindedness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Influence Function</th>
<th>Balance</th>
<th>Monotonicity</th>
<th>Duality</th>
<th>Open-Mindedness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Euler-based</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>qmax</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Aggregation/ Influence Function</th>
<th>Balance</th>
<th>Monotonicity</th>
<th>Duality</th>
<th>Open-Mindedness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum/ qmax</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>
Some Further Readings about Weighted Semantics

• **Attack-only Graphs**

• **Support-only Graphs**

• **Bipolar Graphs**
Gradual Bipolar Argumentation

Computation
Computing Strength Values

\[ i \leftarrow 0 \]

\textbf{FOR} \( a = 1, \ldots, n \)

\[ s^{(i)}(a) = w(a) \]

\textbf{DO}

\[ i \leftarrow i + 1 \]

\textbf{FOR} \( a = 1, \ldots, n \)

\[ s^{(i)}(a) = f(w(a), \text{Parents}(a), s^{(i-1)}) \]

\textbf{UNTIL} \( |s^{(i)} - s^{(i-1)}| < \varepsilon \)

\[ s \leftarrow s^{(i)} \]
**Depth in Acyclic BAGs**

**Depth(i) is defined as**

\[
\begin{align*}
\text{Depth}(i) &= 0 & \text{if Parents}(i) = \emptyset \\
&= 1 + \max \{\text{depth}(j) : j \in \text{Parents}(i)\} & \text{else}
\end{align*}
\]
Convergence in Acyclic BAGs

**Lemma**
If depth(A)=d, then strength of A remains unchanged after iteration d.

**Theorem**
In acyclic BAGs, strength values converge in n-1 iterations.

**Theorem**
Computing strength values once according to topological ordering yields the same result.

- $O(n^2)$ updates
- $O(n+m)$ for ordering + $O(n)$ updates
Convergence in Cyclic BAGs

• In cyclic BAGs, algorithm may not converge (Mossakowski, Neuhaus 2018)

Digression: Lipschitz Continuity

• Lipschitz-continuous: „function does not grow faster than some linear function“

  \[ |f(x_1) - f(x_2)| \leq \lambda \times |x_1 - x_2| \text{ for all } x_1, x_2 \]

• \( \lambda \) is called Lipschitz-constant
Convergence in Cyclic BAGs

• **Sufficient conditions for converge can be derived assuming**
  • *bounded derivatives* (Mossakowski, Neuhaus 2018) or, more general,
  • *Lipschitz-continuity* (AAMAS 2019)

**Theorem (AAMAS 2019)**
If semantics is contractive, that is,
1. aggregation function has Lipschitz-constant $\lambda_1$,
2. influence function has Lipschitz-constant $\lambda_2$,
3. $\lambda_1 \times \lambda_2 < 1$,
then the algorithm is guaranteed to converge.

Convergence up to D digits after $O\left( C(\lambda_1, \lambda_2) \times D \right)$ iterations

## Some Lipschitz Constants

<table>
<thead>
<tr>
<th>Aggregation Function</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product</td>
<td>max. indegree of any argument in BAG</td>
</tr>
<tr>
<td>Sum</td>
<td>max. indegree of any argument in BAG</td>
</tr>
<tr>
<td>Top</td>
<td>( \leq 2 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Influence Function</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear((k))</td>
<td>( \frac{1}{k} \max {w(i), 1 - w(i) : i = 1, \ldots, n} )</td>
</tr>
<tr>
<td>Euler-based</td>
<td>0.25</td>
</tr>
<tr>
<td>qmax((k))</td>
<td>( \frac{1}{k} \max {w(i), 1 - w(i) : i = 1, \ldots, n} )</td>
</tr>
</tbody>
</table>
# Some Convergence Guarantees

<table>
<thead>
<tr>
<th>Semantics</th>
<th>Aggregation</th>
<th>Influence</th>
<th>Sufficient Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Mossakowski, Neuhaus 2018)</td>
<td>Top</td>
<td>Euler-based</td>
<td>Always</td>
</tr>
<tr>
<td>DF-QuAD (k=1)</td>
<td>Product</td>
<td>Linear(k)</td>
<td>Max. indegree &lt; k</td>
</tr>
<tr>
<td>Euler-based</td>
<td>Sum</td>
<td>Euler-based</td>
<td>Max. indegree &lt; 4</td>
</tr>
<tr>
<td>Quadratic Energy (k=1)</td>
<td>Sum</td>
<td>qmax(k)</td>
<td>Max. indegree &lt; k</td>
</tr>
</tbody>
</table>
# Convergence Guarantees vs. Open-Mindedness

## Aggregation Influence $k$ attacks

<table>
<thead>
<tr>
<th>Aggregation</th>
<th>Influence</th>
<th>$k=0$</th>
<th>$k=1$</th>
<th>$k=10$</th>
<th>$k=100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>Euler</td>
<td>0.9</td>
<td>0.862</td>
<td>0.862</td>
<td>0.862</td>
</tr>
<tr>
<td>Addition</td>
<td>Euler</td>
<td>0.9</td>
<td>0.862</td>
<td>0.811</td>
<td>0.811</td>
</tr>
<tr>
<td>Top</td>
<td>qmax(1)</td>
<td>0.9</td>
<td>0.498</td>
<td>0.498</td>
<td>0.498</td>
</tr>
<tr>
<td>Addition</td>
<td>qmax(1)</td>
<td>0.9</td>
<td>0.498</td>
<td>0.012</td>
<td>0.001</td>
</tr>
<tr>
<td>Top</td>
<td>qmax(5)</td>
<td>0.9</td>
<td>0.873</td>
<td>0.873</td>
<td>0.873</td>
</tr>
<tr>
<td>Addition</td>
<td>qmax(5)</td>
<td>0.9</td>
<td>0.873</td>
<td>0.213</td>
<td>0.004</td>
</tr>
</tbody>
</table>
Convergence Guarantees vs. Open-Mindedness

Lemma (AAMAS 2019)
If semantics is defined by
1. aggregation function that maps to [-B, B],
2. combination function with Lipschitz-constant $\lambda$,
then $|s(i) - w(i)| \leq \lambda \times B$.

<table>
<thead>
<tr>
<th>Aggregation Function</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product</td>
<td>[-1, 1]</td>
</tr>
<tr>
<td>Sum</td>
<td>(-\infty, \infty)</td>
</tr>
<tr>
<td>Top</td>
<td>[-1, 1]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Influence Function</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear(k)</td>
<td>$\geq \frac{1}{k}$</td>
</tr>
<tr>
<td>Euler-based</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>qmax(k)</td>
<td>$\geq \frac{1}{k}$</td>
</tr>
</tbody>
</table>

Convergence Guarantees vs. Open-Mindedness

- Constant
- QuadraticEnergy(∞)
- DF-QuAD
- Top/Euler-based
- QuadraticEnergy(N)
- QuadraticEnergy(100)
- QuadraticEnergy(10)

Quadratic Energy
Improving Guarantees by Continuization

• (Discrete) semantics can be seen as coarse approximations of continuous semantics

• Continuizing semantics can solve divergence problems without loosing open-mindedness
Improving Guarantees by Continuization

**Theorem (AAMAS 2019)**
If semantics is contractive (satisfies convergence conditions), continuized semantics converges to the same strength values.

Empirically, convergence in subquadratic time.

Convergence Guarantees for Continuized Semantics

• **Support-only**: yes *(mon. increasing and bounded from above)*

• **Attack-only**: probably *(hand-waving argument)*

• **Bipolar**: maybe *(neither proof idea nor counterexamples are known)*

Some Further Readings about Computational Issues

• **Fixed points in Social Abstract Argumentation**


• **Convergence of Discrete Semantics in Attack-only Graphs**


• **High-Level Introduction to Continuous Semantics**

Gradual Bipolar Argumentation

Programming with Attractor
Attractor

Initialize

Update

Algorithm

Strength Values

Visualizations

Runtime results

https://sourceforge.net/projects/attractorproject/
arg(buy, 0.5).
arg(sell, 0.5).
arg(a1, 0.8).
arg(a2, 0.7).
arg(a3, 0.9).

att(buy, sell).
att(sell, buy).
sup(a1, sell).
sup(a2, buy).
att(a2, a1).
sup(a3, buy).
att(a3, a1).

Buy: 0.5
Sell: 0.5
A2: 0.7
A1: 0.8
A3: 0.9
Solving Weighted Argumentation Problems

```java
AbstractDynamicArgumentationSystem ads = new ContinuousDFQuADModel();

AbstractIterativeApproximator approximator = new RK4(ads);
ads.setApproximator(approximator);

BAGFileUtils fileUtils = new BAGFileUtils();
BAG bag;

try {
    bag = fileUtils.readBAGFromFile(new File("files/PresentationBAG.bag"));
    ads.setBag(bag);
    ads.approximateSolution(10e-3, 10e-4, true);
} catch (Exception e) {
    e.printStackTrace();
}
```
ads.setApproximator(new PlottingRK4(ads));
ads.approximateSolution(10e-3, 10e-4, true);

Generate visualizations with JFreeChart
ads.setApproximator(new PlottingEulersMethod(ads));
ads.approximateSolution(1, 10e-4, true);

Simulate Discrete Semantics using Euler’s Method (KR 2018)

Evolution Continuous DF-QuAD Model, Euler’s Method, d=1.0, e=0.001

BenchmarkUtils benchmark = new BenchmarkUtils();
File benchmarkDirectory = new File("files/networks/barabasi");
QuadraticEnergyModel qas = new QuadraticEnergyModel();

benchmark.runBenchmark(benchmarkDirectory, qas);
Using and Adding Semantics

AbstractDynamicArgumentationSystem

- ContinuousDFQuadModel
- ContinuousEulerBasedModel
- ContinuousModularModel
  - AggregationFunction
    - ProductAggregation
    - SumAggregation
  - Influence Function
    - LinearInfluence
    - EulerBasedInfluence
  - ...

...
Using and Adding Algorithms

AbstractIterativeApproximator

Euler's Method

RK4 (recommended)

AbstractPlottingIterativeApproximator

PlottingEuler's Method

PlottingRK4 (recommended)
Documentation

• **Tutorial Article**


  

• **Javadoc**

  https://sourceforge.net/projects/attractorproject/
Summary
Quantitative AAFs assign numerical values (belief, strength) to arguments. Values can be explained by going backwards through the graph. Rules can be used to generate simple automated explanation dialogues.
Summary

- Probabilistic Epistemic Argumentation
  - Evaluation: probabilities
  - Complexity: (polynomial)
  - Model
    - Bipolar Argumentation Graph
    - Semantical Constraints
  - Implementation
    https://sourceforge.net/projects/probabble/

- Gradual Bipolar Argumentation
  - Evaluation: strength values
  - Complexity: (polynomial)
  - Model
    - Bipolar Argumentation Graph
    - Initial Weights
    - Update function
  - Implementation
    https://sourceforge.net/projects/attractorproject/
